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ON THE OPTIMUM DESIGNOF CATHODE FOLLOWERS

by
L. M. VALLESE
Report R-390-54, PIB-323

for
OFFICE OF NAVAL RESEARCH
Contract No. Nonr-839(05)
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ON THE OPTIMUM DESIGN OF CATHODE FOLLOWERS

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L. M. Vallese

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Brooklyn 1, New York
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ABSTRACT

An analytical procedure to obtain a preliminary design of large signal cathode follower amplifiers is described. A-C amplifiers with conductive and inductive cathode admittance as well as d-c amplifiers are considered. The conditions of optimization of the design for maximum power or voltage output are discussed.

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Introduction

Cathode followers find wide application in modern Electronics because of their simple structure, fairly distortionless operation and low output impedance. Their design for small signal amplification is easy; for large signal amplification an added complication is introduced with the requirement that the distortion-free driving range of the control grid be utilized fully. Usually this problem is solved graphically, but the procedure is cumbersome and of limited application. Its usefulness can be greatly enhanced if it is used merely to check and improve an approximate preliminary design obtained analytically. The derivation of such a preliminary design is the object of this report. For this purpose the family of characteristics of the tube will be represented by a bilinear relationship

$$i_b = g_p (e_b - v_{bk}) = g_p (e_b + \mu e_{ck} - V_o^{\dagger})$$
 (1)

which corresponds to a set of straight lines (Fig. MRI-ll276-a) characterized by their slope g_p and their intercept V_{bk} with the i_b = 0 axis (in particular V_c and V_{bk} are respectively the intercepts corresponding to E_{ck} = 0 and E_{ck} = -0.5 V). The main purpose of the design consists in determining the values of E_{0} , In so that the grid-to-cathode voltage may be varied between -0.5 V and a value corresponding to a minimum current I_{bm} . In addition the design may be optimized with respect to the power and voltage output, plate dissipation, etc. We shall discuss separately the cases of a-c and d-c amplifiers and shall show that the design of the a-c amplifiers is subject to less restrictions than that of d-c amplifiers. The cathode admittance of a-c amplifiers can be either a conductance $G_{\overline{K}}$ or a (small) inductive susceptance $1/j\omega L$ = - $jE_{\overline{K}}$, while the cathode admittance of d-c amplifiers must be a conductance $G_{\overline{K}}$.

I. A-C Cathoda Followers with Conductive Cathoda Admittance

The circuit diagram of a typical a-c cathode follower with conductive cathode admittance is indicated in Fig. MRI-lh276-b. We shall disregard high and low frequency effects and limit our analysis to the mid-frequency range. For this purpose the interelectrode and coupling capacitances will be neglected, and the design will be limited to the determination of the quantities GK, GL, Ebb, Eo, Es, (Eck)g, Eg, Ig. The interpretation of these symbols is clear from Fig. MRI-lh276-b; we shall only point out that Eo and Es are peak rather than rms values.

The following limitations must be satisfied:

$$E_{ck} \leq E_{ck \text{ max}} = 0.5 \text{ V}$$

$$I_{bm} \leq i_b \leq I_{k \text{ max}}$$

$$E_{bb} \leq E_{bb \text{ max}}$$

$$E_{\mathbf{Q}} = I_{\mathbf{Q}} \leq I_{p \text{ max}}$$
(2)

The last of conditions (2) applies for zero signal dissipation; it could be replaced with the less restrictive condition Eq Iq - R_{DMex} , if the power output PL were a constant.

The following relationships can be used for the analysis:

$$2I_{\mathbf{Q}} = I_{\mathbf{bm}} + I_{\mathbf{bM}} \tag{3}$$

$$I_{bM} = g_{p} (E_{Q} - E_{o} - V_{bk}) = g_{p} (e_{bm} - V_{bk})$$
 (14)

$$I_{C_{\parallel}} = G_{k} (E_{bb} - E_{Q}) = I_{bm} + E_{o} (G_{k} + G_{L})$$
 (5)

$$E_Q = e_{bm} + E_o = V_{bk}^i + \frac{I_{bk}}{g_p} + E_o$$
 (6)

$$\mathbf{E}_{bb} = \mathbf{E}_{\mathbf{Q}} + \mathbf{I}_{\mathbf{Q}}/\mathbf{Q}_{\mathbf{k}} \tag{7}$$

$$(E_{ck})_{Q} = \frac{1}{\mu} (V_{o}' - V_{bk}) = \frac{1}{\mu} (V_{o}' - E_{Q} + I_{Q}/E_{p})$$
 (8)

$$\mathbf{E}_{\mathbf{S}} = \mathbf{E}_{\mathbf{C}} + (\mathbf{E}_{\mathbf{C}\mathbf{k}})_{\mathbf{Q}} = 0.5 \tag{9}$$

$$e_{bm} = V_{bk}^{i} + \left[I_{bm} + 2 E_{o} (G_{k} + G_{L}) \right] / g_{p}$$
 (10)

By elimination of E_{Q} , I_{Q} from (3),(4),(5) one has (11)

$$G_{k}^{2} - \frac{g_{p}}{2E_{0}} \left[E_{bb}^{2} - 2E_{o}(1 + \frac{G_{L}}{g_{p}}) - \frac{I_{bm}}{g_{p}} - V_{bk}^{i} \right] G_{k} + \frac{g_{p}}{2} (G_{L} + \frac{I_{bm}}{E_{o}}) = 0$$

which is a relationship among $E_{\rm bb}$, $E_{\rm o}$, $G_{\rm k}$, $G_{\rm L}$ and can be used to determine one of these quantities when the other three are given. One has:

$$G_{\underline{I}} = \frac{g_{\underline{p}} \left[\underline{E}_{\underline{b}\underline{b}} - \underline{V}_{\underline{b}\underline{k}}^{\dagger} - 2\underline{E}_{\underline{o}}^{\dagger} G_{\underline{k}}^{\dagger} - 2\underline{E}_{\underline{o}}^{\dagger} G_{\underline{k}}^{2} - \underline{I}_{\underline{b}\underline{m}} (g_{\underline{p}} + G_{\underline{k}}) \right]}{\underline{E}_{\underline{o}} (2 G_{\underline{k}} + g_{\underline{p}})}$$
(12)

$$E_{o} = \frac{g_{p}(E_{bb} - V_{bk}^{*}) G_{k} - (g_{p} + G_{k}) I_{bm}}{2 G_{k}(G_{k} + g_{p}) + (g_{p} + 2 G_{k}) G_{L}}$$
(13)

or
$$G_{k} = \frac{g_{p}}{4E_{o}} \left[E_{bb} - 2E_{o}(1 + \frac{G_{L}}{g_{p}}) - \frac{I_{bm}}{g_{p}} - V_{bk} \right] + \sqrt{\frac{G^{2}}{6E_{o}^{2}}} \right]^{2} - \frac{g_{p}}{2} \left(G_{L} + \frac{I_{bm}}{E_{o}} \right)$$

and finally

$$E_{bb} = V_{bk}' + I_{bm} \left(\frac{1}{Q_k} + \frac{1}{g_p} \right) + 2E_o \left(1 + \frac{Q_L}{2Q_k} + \frac{Q_k + Q_L}{g_p} \right)$$
 (35)

Two other important relations are ;

$$I_{bM} = I_{bm} + 2E_{o}(G_{k} + G_{L}) = \frac{2(G_{k} + G_{L})g_{p} G_{k}(E_{bb} - V_{bk}^{\dagger}) - g_{p}G_{L}I_{bm}}{2(g_{p} + G_{k})(G_{k} + G_{L}) - g_{p}G_{L}}$$
(16)

and

$$P_{p} = E_{\mathbf{Q}^{\mathbf{I}}\mathbf{Q}} = \left[E_{o}(1 + \frac{2(G_{k} + G_{\mathbf{L}})}{g_{p}}) + V_{bk}^{i} + \frac{I_{bm}}{g_{p}}\right]\left[I_{bm} + E_{o}(G_{k} + G_{\mathbf{L}})\right] =$$

$$= \left[\mathbb{E}_{bb} - \frac{1}{\mathbb{Q}_{k}} \left(\mathbb{I}_{bm} + (\mathbb{Q}_{k} + \mathbb{Q}_{L}) \mathbb{E}_{o} \right) \right] \left[\mathbb{I}_{bm} + \mathbb{E}_{o} \left(\mathbb{Q}_{k} + \mathbb{Q}_{L} \right) \right]$$
(17)

We shall now proceed to consider a number of specific problems that can occur in practice, and derive the optimum approximate solution. In particular, numerical design values of somewhat academic interest will be computed, assuming that the tube employed is the 6J5, whose plate characteristics are shown in Fig. MRI-1h277. The latter ones are linearized assuming $g_p = 1/7600 = 1.32 \times 10^{-14}$ mhos, $V_0^* = 30V$, $V_{DK}^* = 10 V$, $I_{DK}^* = 0.5$ mA, $\mu = 20$, $I_{KM}^* = 20$ mA, $E_{DDM}^* = 300$ V, $P_{DMX}^* = 2.5$ W.

Design No. 1 - Given the tube, G_K , G_L , E_O find the minimum necessary value of $E_{\rm bh}$.

One uses formula (15), then checks that the values of $E_{\rm bb}$, $I_{\rm bM}$, $P_{\rm b}$ are less than the limiting values (2). Finally one computes $(E_{\rm ck})_{\rm Q}$, $E_{\rm GG}$ and $E_{\rm s}$.

For example assume that the tube is a 6J5 and that $G_k = 4 \times 10^{-4}$ mhos, $G_L = 10^{-3}$ mhos, $E_0 = 5$ V. There follows:

$$E_{bb} = 40 + 0.5 \times 10^{-3} (2500 + 7600) + 10 (12.85) = 173.5 \text{ V}$$

The check with respect to the limit performance of the tube provides

$$I_{bM} = 14.5 \text{ mA}$$
 $I_{Q} = 7.5 \text{ mA}$
 $E_{Q} = 116 \text{ V}$
 $P_{p} = 0.87 \text{ W}$

Furthermore

$$(E_{ck})_{Q} = \frac{1}{20} [30-116+57] = -1.45 \text{ V}$$

$$E_{GG} = E_{Q} + (E_{ck})_{Q} = 114.5 \text{ V}$$

$$E_{g} = 5 + 1.45 - 0.5 = 5.95 \text{ V}$$

$$K = \frac{E_{o}}{E_{e}} = 0.84$$

Design No. 2 - Given the tube, Og, GL, find Ebb for max. Eo permissible.

Since E_0 increases linearly with E_{bb} , we shall merely investigate the maximum value of E_0 . With respect to the current limitation this is (eq. 16)

$$E_{o}' = \frac{I_{kM} - I_{cm}}{2(G_{k} + G_{L})}$$
 (18)

With respect to the power limitation one has from eq. 17

$$\mathbb{E}_0^{\pi} = - \prec + \sqrt{\prec^2 + \beta} \tag{19}$$

where

$$= \frac{\left[g_{p} + 2(G_{k} + G_{L})\right] I_{bm} + (G_{k} + G_{L})(g_{p} V_{bk} + I_{bm})}{2(G_{k} + G_{L})\left[g_{p} + 2(G_{k} + G_{L})\right]}$$

$$\beta = \frac{g_{p} P_{pmax} - (g_{p} V_{bk}^{i} + I_{bm}) I_{bm}}{(G_{k} + G_{L})(g_{p} + 2(G_{k} + G_{L}))}$$

The lower of the values (18), (19) is the maximum permissible E_0 . Correspondingly $E_{\rm bb}$ is obtained from eq. 15. For example, for a 6J5 with $G_{\rm k}$ = μ x 10-4 mhos, $G_{\rm L}$ = 10-3 mhos one has

$$E_0^* = 7.3 \text{ V}$$
, $E_0^* = 7.83 \text{ V}$

The maximum permissible value $E_{\rm o}$ is 7.3 V and the corresponding value of $E_{\rm bb}$ is 232.5 V,

Design No. 3 - Given the tube, GL, Eo, Rbb find Gk.

One uses eq. (14) which provides two solutions for G_k . These are checked with respect to conditions (2). Assuming that both G_k values are permissible, the final choice will be determined by considerations of the plate power dissipation, peak cathode current, output admittance.

For example for a 6J5, given $G_L = 10^{-3}$ mhos, $E_0 = 5V$, $E_{bb} = 173$ V, the corresponding values of G_k are

$$a_{k}^{\dagger} = 0.4 \times 10^{-3} \text{ mhos}$$
 $a_{k}^{\dagger} = 0.177 \times 10^{-3} \text{ mhos}$ (20)

Incidentally, if Ebb & 200 V the valuer of Gk are

$$a_{k}^{\dagger} = 0.837 \times 10^{-3} \text{ mhos}$$
 $a_{k}^{\dagger} = 0.083 \times 10^{-3} \text{ mhos}$

In connection with the values (20) one finds

$$I_{bM}^{'} = 14.5 \text{ mA}$$
 $I_{bM}^{''} = 12.27 \text{ mA}$ $E_{Q}^{''} = 116 \text{ V}$ $E_{Q}^{''} = 98.2 \text{ V}$ $P_{D}^{''} = 0.625 \text{ W}$

Design No. h - Given the tube, G_L , E_{bb} find G_k for max. E_o . From eq. (13), letting $\frac{dE_o}{dG_k}$ = 0, one has

$$a_{k} = \frac{g_{p} I_{bm}}{g_{p} (E_{bb} - V_{bk}) - I_{bm}} + \sqrt{\frac{g_{p}^{2} I_{bm}^{2}}{\left[g_{p} (E_{bb} - V_{bk}) - I_{bm}\right]^{2}} + \frac{g_{p}}{2} (a_{L} + \frac{2(g_{p} + a_{L}) I_{bm}}{g_{p} (E_{bb} - V_{bk}) - I_{bm}})}$$

For example, letting $G_L = 10^{-3}$ mhos, $E_{\rm bb} = 173$ V, one has for the 6J5

and from eq. (13)

For given G_L and E_{bb} it is of interest to compute the limit values of G_K for which respectively $I_{bM} = I_{kM}$ or $P_p = P_{pmax}$. By application of eq. (13) these provide the corresponding limit values of E_0 .

For the condition $I_{bM} = I_{kM}$ one has from eq. 16, solving for G_k

$$G_{k} = -b + \sqrt{b^{2} - c}$$
 (22)

where

$$b = -\frac{G_L}{2} + \frac{g_p I_{kM}}{2 g_p (E_{bb} - V_{bk}^i) - 2I_{kM}}$$

$$c = \frac{g_p G_L \left[I_{kM} - I_{bm}\right]}{2 g_p \left(E_{bb} - V_{bk}\right) - 2 I_{kM}}$$

In particular, if Gt = 0, eq. (22) reduces to

$$\frac{C_{k}}{I_{kM}} = \frac{1}{g_{p}}$$
 (23)

Similarly eqs. (13) and (17) provide the limit values G_k and E_0 for which $P_p = P_{pmax}$. Since these eqs. are of the fifth degree in G_k , they should be solved by trial and error method.

Design No. 5 - Given the tube, $E_{\rm bb}$, find $G_{\rm k}$ and $G_{\rm L}$ for max. output power $F_{\rm L} = G_{\rm L} E_{\rm c}^2/2$. This design may be considered of academic interest, since cathode followers for power amplification are usually made with inductive rather than conductive cathode admittance.

According to eq. 13 one has

$$P_{L} = \frac{G_{L}}{2} \left[\frac{g_{p}(E_{bb} - W_{bk}^{i})G_{k} - (g_{p} + G_{k}) I_{bm}}{2 G_{k} (G_{k} + g_{p}) + (g_{p} + 2G_{k})G_{L}} \right]^{2}$$
(24)

which is a function of $E_{\rm bb}$, $G_{\rm k}$, $G_{\rm L}$. If either $G_{\rm k}$ or $G_{\rm L}$ are given, the optimum value of the conductances $G_{\rm L}$ or $G_{\rm k}$ is determined respectively with the condition

$$\frac{dP_{\underline{L}}}{dG_{\underline{L}}} = 0 \quad \text{or} \quad \frac{dP_{\underline{L}}}{dG_{\underline{k}}} = 0 \tag{25}$$

If G_k and G_l are both indetermined, the latter two equations must be solved simultaneously.

From eq. 25 there follows

$$G_{L} (2G_{k} + g_{p}) - 2G_{k} (G_{k} + g_{p}) = 0$$
 (26)

$$G_{k}^{2} + \frac{g_{p} I_{bm} G_{k}}{g_{p} (E_{bb} - V_{bk}) - I_{bm}} - \frac{g_{p}}{2} \left[G_{L} + \frac{2(g_{p} + G_{L}) I_{bm}}{g_{p} (E_{bb} - V_{bk}) - I_{bm}} \right] = 0$$

In particular the last equation for $I_{bm} = 0$ simplifies as follows

$$2 g_{k}^{2} - g_{p} g_{L} = 0 (27)$$

Therefore, if Gk is given, the optimum value of GL is

$$G_{L} = 2 G_{k} \frac{G_{k} + g_{p}}{2G_{k} + g_{p}}$$
 (28)

If G_L is given the optimum value of G_k is computed by means of eq. 21 and in particular for $I_{\rm bm}$ = 0 one has

$$G_{k} = \sqrt{g_{p} G_{L}/2}$$
 (29)

Finally, if both $G_{\mathbf{k}}$ and $G_{\mathbf{L}}$ are indetermined, their optimum values are respectively

$$G_k = g_p / \sqrt{2}$$
, $G_L = g_p$ (30)

Correspondingly the peak power output is

$$(P_{L})_{peak} = \frac{0.0215}{2} g_{p} \left[E_{bb} - V_{bk}^{\prime}\right]^{2}$$

For example, with a 6J5 with $E_{\rm bb} = 200 \mbox{V}(P_{\rm L})_{\rm peak}$ is 0.363 w. In Table I the quantity $2P_{\rm L}/g_{\rm p}$ ($E_{\rm bb} - V_{\rm bk}^{\rm i}$)² has been computed for various values of $G_{\rm k}/g_{\rm p}$, $G_{\rm L}/g_{\rm p}$ near the optimum values (30).

Table I - Values of
$$2P_L \left(\mathbf{x}_{bb} - \mathbf{v}_{bk}^{\dagger} \right)^2$$

G _k	g _p /√2	g _p	√2 e _p
g _p /2	0.0208	0.0204	0,0188
$g_p/\sqrt{2}$	0.0208	0.0215	0.0208
$\mathbf{z}_{\mathbf{p}}$	0.0188	0.02011	0.0208

II. A-C Cathode Followers with Inductive Cathode Admittance

The circuit diagram of a typical cathode follower with inductive cathode admittance is indicated in Fig. MRI-lh278-a. It is assumed that $jB_{\rm K}=1/j\omega L$ is negligible; in addition, high and low frequency effects are neglected.

The limitations of the design are expressed by equations (2), where $E_Q = E_{\rm DD}$. The design itself is much simplified in the present case and reduces to finding I_Q , since E_Q is fixed. The design equations are

$$I_{\mathbf{Q}} = I_{\mathbf{bm}} + G_{\mathbf{L}} E_{\mathbf{o}}$$
 (31.1)

$$I_{bM} = g_{p} (E_{bb} - V_{bk}^{!} - E_{o}) \approx 2 I_{Q} - I_{bm}$$
 (31.2)

$$(\mathbf{E}_{ck})_{\mathbf{Q}} = \left[\mathbf{V}_{c}^{\dagger} - \mathbf{E}_{bb} + \mathbf{I}_{\mathbf{Q}} / \mathbf{g}_{p} \right] / \mu$$
 (31.3)

$$\mathbf{E}_{\mathbf{g}} = \mathbf{E}_{\mathbf{o}} + (\mathbf{E}_{\mathbf{o}k})_{\mathbf{o}} - 0.5 \tag{31.4}$$

Eliminating Ic between first and second equation one has

$$g_{p}(E_{bb} - V_{bk}') - I_{bm} = E_{o}(2 G_{L} + g_{p})$$

From this equation, if $E_{\mbox{\scriptsize bb}}$ and $G_{\mbox{\scriptsize L}}$ are given, one has

$$E_{o} = \frac{g_{p}(E_{bb} - V_{bk}') - I_{bm}}{2 G_{L} + g_{p}}$$
(32)

and if Ebb and Eo are given one has

$$G_{L} = \frac{g_{p} (E_{bb} - V_{bk} - E_{o}) - I_{Dill}}{2 E_{o}}$$
 (33)

The maximum possible value of $\mathbf{E}_{\mathbf{0}}$ for given $\mathbf{E}_{\mathbf{bb}}$ occurs when $\mathbf{G}_{\mathbf{L}}$ = 0 and is

$$(E_o)_{max} = E_{bb} - V_{bk}^i - I_{bm}/g_p.$$
 (34)

On the other hand, when $G_{\mathbf{L}} \neq 0$, the maximum possible value of $\mathbf{E}_{\mathbf{O}}$ is

$$(E_0)_{\text{max}} = (I_{\text{kM}} - I_{\text{bm}})/2 G_{\text{L}}$$
 (35)

according to eqs. (31.2) and (32).

As an example, given a 6J5 with $E_{\rm bb}$ = 200 V, $G_{\rm L}$ = 10^{-3} mhos there follows

$$E_0 = 9.65 \text{ V}$$
 $I_Q = 10.15 \text{ mA}$
 $(E_{Gk})_Q = -9.7 \text{ V}$

The maximum possible value of E_0 for the given value of G_L is 9.75 V; the maximum value of E_0 for G_L = 0 is 156.2 V.

An interesting problem is the determination of the optimum value of G_{T} for maximum power output. One has

$$P_{L} = G_{L} E_{C}^{2}/2 = \frac{G_{L}}{2 \left[2G_{L} + g_{D}\right]^{2}} \left[g_{p} \left(E_{bb} - V_{bk}^{t}\right) - I_{bm}\right]^{2}$$

and letting $\frac{dP_L}{dG_L} = 0$ there follows

$$2 G_{\mathbf{L}} = g_{\mathbf{p}} \tag{36}$$

which is the same result that applies for grounded cathode amplifiers.

III. D-C Cathode Followers

A typical circuit for the d-c cathode follower amplifier is indicated in Fig. MRI-14278-b. For generality it has been assumed that E_{bb}^{\dagger} \neq E_{bb}^{*} .

One of the conditions of the design is that the cathode be at ground petential. Correspondingly one has

$$\mathbf{E}_{\mathbf{Q}} = \mathbf{E}_{\mathbf{bb}}^{'} \qquad \mathbf{I}_{\mathbf{Q}} = \mathbf{G}_{\mathbf{k}} \mathbf{E}_{\mathbf{bb}}^{''} \qquad (37)$$

By application of Thevenin theorem the circuit of Fig. MRI-14278-b can be modified into that of Fig. MRI-14279a, with

$$G' = G_k + G_L, \quad E' = \frac{G_k E_{bb}^{N}}{G_k + G_L}$$
 (38)

The latter circuit is equivalent to that of Fig. MRI-14279b which is similar to the circuit of an a-c cathode follower having zero load admittance G_{τ} .

The limit conditions of operation are

$$E_{ck} \leq E_{ck \text{ max}} = -0.5 \text{ V}$$

$$I_{Q} = 0_{k}^{+} E_{bb}^{N} \leq (I_{kM} + I_{bm}) / 2$$
(39)

$$P_{p} = \mathcal{I}_{bb}^{\dagger} I_{\mathbf{Q}} \approx P_{p \text{ max}} \tag{40}$$

The design equations are

$$E_{bb}^{i} = E_{c} + V_{bk}^{i} + \frac{I_{bm}}{g_{p}} = V_{bk}^{i} + E_{c} \left(1 + \frac{2(G_{k} + G_{L})}{g_{p}}\right)$$
 (41)

$$I_{\mathbf{Q}} = \frac{1}{2} \left[I_{bm} + g_{\mathbf{p}} \left[E_{bb}' - V_{bk}' - E_{\mathbf{o}} \right] \right]$$
 (42)

$$I_{\mathbf{Q}} = I_{bm} + E_{\mathbf{o}} (G_{\mathbf{k}} + G_{\mathbf{L}})$$
 (43)

$$(\mathbf{E}_{ck})_{\mathbf{Q}} = \begin{bmatrix} \mathbf{V}_{o}^{'} - \mathbf{E}_{bb}^{'} + \mathbf{I}_{\mathbf{Q}}/\mathbf{g}_{p} \end{bmatrix} / \mu \tag{114}$$

$$\mathbf{E}_{\mathbf{s}} = \mathbf{E}_{\mathbf{o}} \div (\mathbf{E}_{\mathbf{ok}})_{\mathbf{a}} \sim 0.5 \tag{45}$$

In general E_0 and G_1 are prescribed and the purpose of the design is to choose a convenient tube and evaluate E_{bb}^{\dagger} , E_{bb}^{η} and G_{K}^{\dagger} .

The choice of the tube is determined by the maximum voltage output $E_{\rm o}$ that it can deliver in accordance with the limitations (39) and (40). The upper limit of $E_{\rm o}$ is obtained letting $G_{\rm k}$ approach zero. Combining eqs. (39) and (43) one finds

$$\left(\mathbf{E}_{o}^{'}\right)_{\text{max}} = \frac{\mathbf{I}_{\text{kM}} - \mathbf{I}_{\text{bm}}}{2\mathbf{G}_{\text{T}}} \tag{46}$$

On the other hand combining eqs. (40), (41) and (43) one has

$$\left\{ \mathbb{E}_{o} \left(1 + \frac{2(G_{k} + G_{L})}{g_{p}} \right) + \mathbb{V}_{bk}^{\dagger} \right\} \left[\mathbb{E}_{o} \left(G_{k} + G_{L} \right) + \mathbb{I}_{bm} \right] \leqslant P_{pM}$$

wherefrom there follows

$$(E_o^{"})_{\text{max}} \approx \frac{-V_{\text{bk}}}{2(1 + \frac{2G_L}{g_D})} + \sqrt{\frac{g_p P_{\text{pM}}}{G_L(g_p + 2G_L)}}$$
 (47)

The lower of the values (46) and (47) should be larger than the required \mathbf{E}_{o} .

After the tube has been selected one proceeds to evaluate $E_{\rm bb}^{\dagger}$. The upper limit of $E_{\rm bb}^{\dagger}$ again is determined in accordance to the limitations (39) and (40). Combining eqs. (39) and (42) one has

$$E_{bb}' \leq \frac{I_{kM} - I_{bm}}{\varepsilon_{p}} + V_{bk}' + E_{o}$$
 (48)

On the other hand, combining eqs. (40) and (42) one has

$$E_{bb}^{'} \leq \frac{1}{2} \left(v_{bk}^{'} + E_{o} - \frac{2I_{bm}}{g_{p}} \right) + \sqrt{\frac{1}{l_{l}} \left(v_{bk}^{'} + E_{o} - \frac{2I_{bm}}{g_{p}} \right)^{2} + \frac{2P_{pM}}{g_{p}}}$$
 (49)

In general the value of $E_{\rm bb}$ is chosen in dependence of the available power supply, so that the limitations (48) and (49) are satisfied.

After the value of $E_{\rm bb}^{\rm i}$ has been determined, eqs. (42) and (43) provide respectively IQ and $G_{\rm K}$. Finally the value of $E_{\rm bb}^{\rm i}$ is obtained from eq. (37) and the values of $(E_{\rm ck})_{\rm Q}$ and $E_{\rm b}$ are computed from eqs. (44) and (45) respectively.

For example let us consider the design of a d-c cathode follower with $E_0=8$ V, $G_L=10^{-3}$ mhos. For the tube 6J5 eqs. (46) and (47) provide respectively

$$(E_o^1)_{max} = 9.75 \text{ V}$$
 $(E_o^N)_{max} = 11.17 \text{ V}$

Therefore the 6J5 is suitable for the present design. According to eqs. (h8) and (h9) the upper limit of E_{bb}^{i} is the lower of the following ones:

$$E_{bb}^{'} \le 196 \text{ V}$$
 $E_{bb}^{'} \le 21h \text{ V}$

Let E = 180 V. Equation (42) now provides

Eq. (43) then gives

$$G_k = 0.06 \times 10^{-3} \text{ mhos}$$

Finally

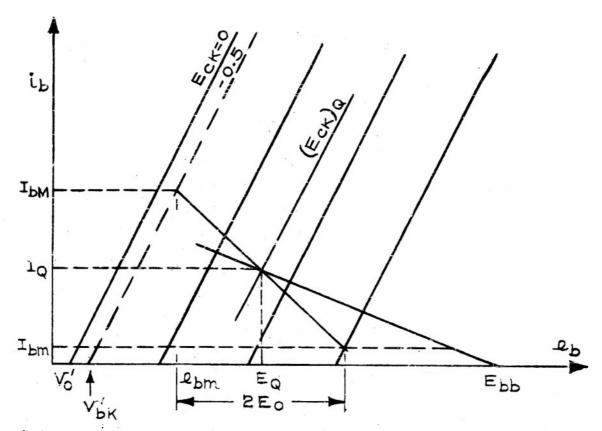
$$E_{bb}^{n} = \frac{I_{Q}}{Q_{k}} = .150 \text{ V}$$

and

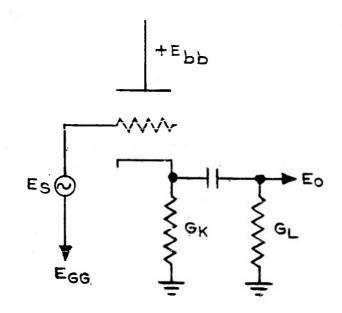
$$-(E_{ok})_{Q} = -4.0 \text{ V}, E_{s} = 11.5 \text{ V}$$

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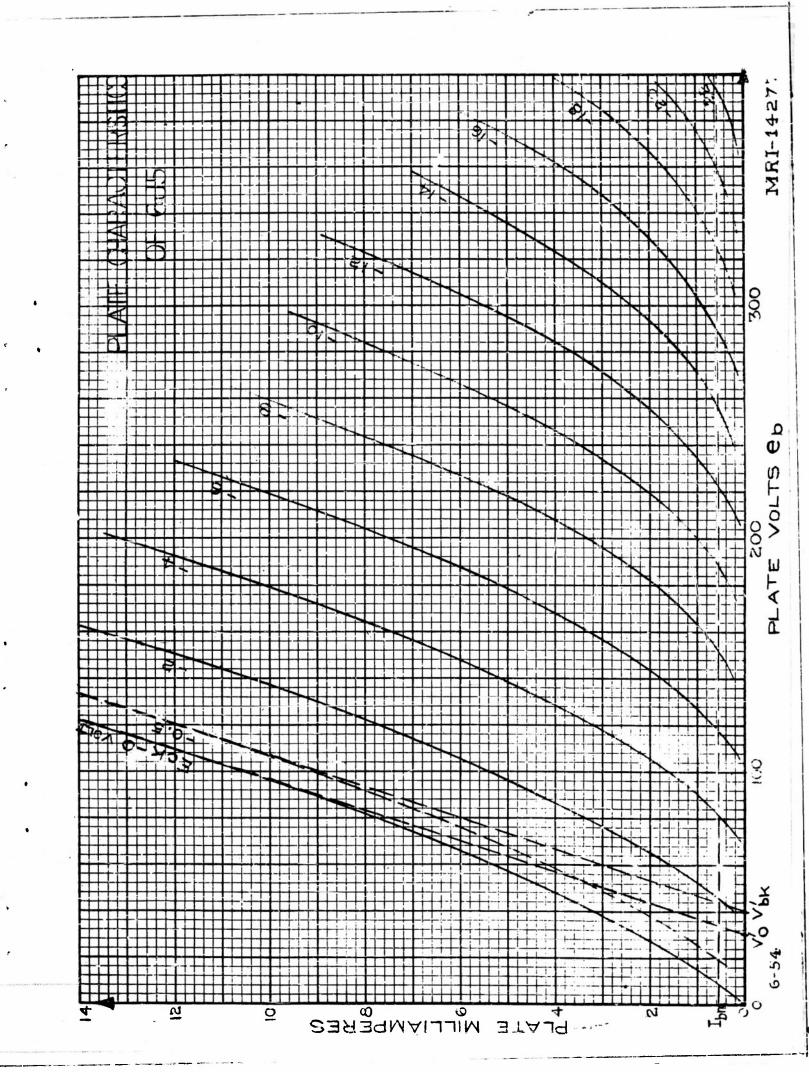
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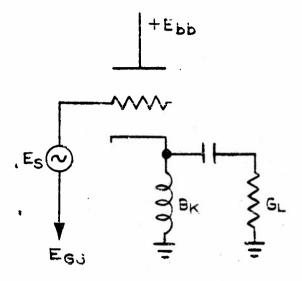
(a)-LINEARIZED PLATE CHARACTERISTICS OF A TRIODE VACUUM TUBE

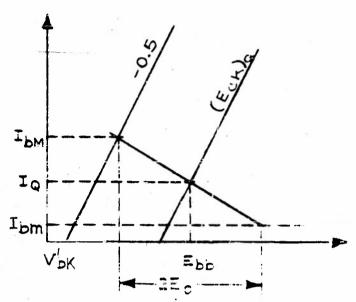


(b)- A-C CATHODE FOLLOWER WITH CONDUCTIVE CATHODE ADMITTANCE

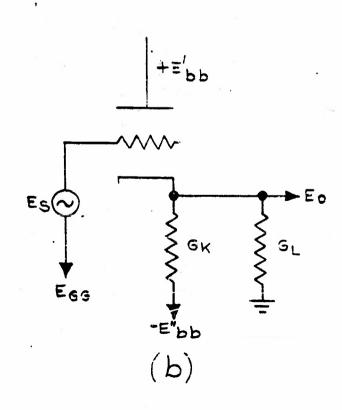


A-C CATHODE FOLLOWER WITH SUSCEPTIVE CATHODE ADMITTANCE

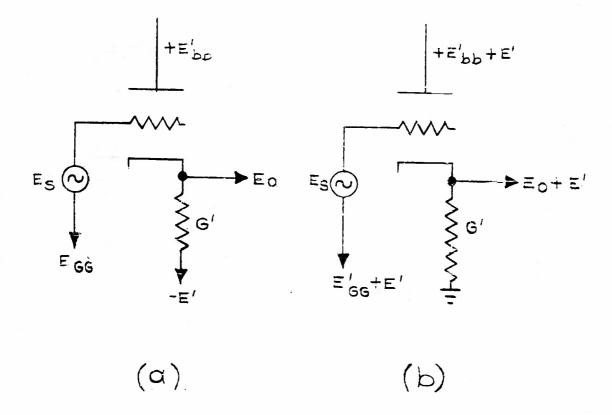




(a)



D-C CATHODE FOLLOWER



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